Quantitative Data Analysis

Summarizing Data: variables; simple statistics; effect statistics and statistical models; complex models.

Generalizing from Sample to Population: precision of estimate, confidence limits, statistical significance, p value, errors.

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Reference: Hopkins WG (2002). Quantitative data analysis (Slideshow). Sportscience 6, sportsci.org/jour/0201/Quantitative_analysis.ppt (2046words)

Summarizing Data

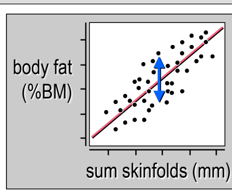
- Data are a bunch of values of one or more variables.
- A variable is something that has different values.
  - Values can be numbers or names, depending on the variable:
    - **Numeric**, e.g. weight
    - **Counting**, e.g. number of injuries
    - **Ordinal**, e.g. competitive level (values are numbers/names)
    - **Nominal**, e.g. sex (values are names
  - When values are numbers, visualize the distribution of all values in stem and leaf plots or in a frequency histogram.
    - Can also use **normal probability plots** to visualize how well the values fit a normal distribution.
  - When values are names, visualize the frequency of each value with a pie chart or a just a list of values and frequencies.

- A statistic is a number summarizing a bunch of values.
  - Simple or univariate statistics summarize values of one variable.
  - Effect or outcome statistics summarize the relationship between values of two or more variables.
- Simple statistics for numeric variables…
  - Mean: the average
  - Standard deviation: the typical variation
  - Standard error of the mean: the typical variation in the mean with repeated sampling
    - **Multiply by √(sample size)** to convert to standard deviation.
  - Use these also for counting and ordinal variables.
  - Use median (middle value or 50th percentile) and quartiles (25th and 75th percentiles) for grossly non-normally distributed data.
  - Summarize these and other simple statistics visually with box and whisker plots.

- Simple statistics for nominal variables
  - Frequencies, proportions, or odds.
  - Can also use these for ordinal variables.
- Effect statistics…
  - Derived from statistical model (equation) of the form Y (dependent) vs X (predictor or independent).
  - Depend on type of Y and X . Main ones:

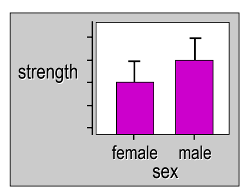
Y	X	Model/Test	Effect statistics
numeric	numeric	regression	slope, intercept, correlation
numeric	nominal	t test, ANOVA	mean difference
nominal	nominal	chi-square	frequency difference or ratio
nominal	numeric	categorical	frequency ratio per…

- Model: numeric vs numeric e.g. body fat vs sum of skinfolds



  - Model or test: linear regression
  - Effect statistics:
    - slope and intercept = parameters
    - correlation coefficient or variance explained (= 100·correlation^2) = measures of goodness of fit
  - Other statistics:
    - typical or standard error of the estimate = residual error = best measure of validity (with criterion variable on the Y axis)

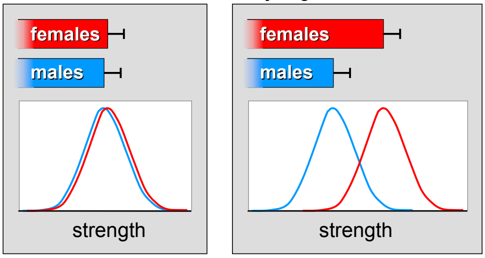
- Model: numeric vs nominal e.g. strength vs sex



  - Model or test:
    - t test (2 groups)
    - 1-way ANOVA (>2 groups)
  - Effect statistics:
    - difference between means expressed as raw difference, percent difference, or fraction of the root mean square error (Cohen's effect-size statistic)
    - variance explained or better √(variance explained/100) = measures of goodness of fit
  - Other statistics:
    - root mean square error = average standard deviation of the two groups

- More on expressing the magnitude of the effect
  - What often matters is the difference between means relative to the standard deviation:

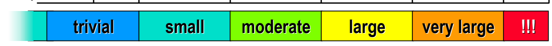
Trivial effect: **Very large effect:**



- Fraction or multiple of a standard deviation is known as the effect-size statistic (or Cohen's "d").
- Cohen suggested thresholds for correlations and effect sizes.
- Hopkins agrees with the thresholds for correlations but suggests others for the effect size:

Correlations

Cohen: 0 0.1 0.3 0.5
Hopkins: 0 0.1 0.3 0.5 0.7 0.9 1

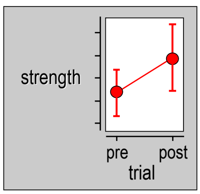


Effect Sizes

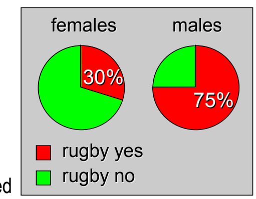
Cohen: 0 0.2 0.5 0.8
Hopkins: 0 0.2 0.6 1.2 2.0 4.0 ∞

- For studies of athletic performance, percent differences or changes in the mean are better than Cohen effect sizes.

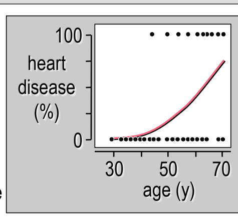
- Model: numeric vs nominal
  (repeated measures)
  e.g. strength vs trial
  - Model or test:
    - paired t test (2 trials)
    - repeated-measures ANOVA with one within-subject factor (>2 trials)
  - Effect statistics:
    - change in mean expressed as raw change, percent change, or fraction of the pre standard deviation
  - Other statistics:
    - within-subject standard deviation (not visible on above plot) = typical error: conveys error of measurement
      - useful to gauge reliability, individual responses, and magnitude of effects (for measures of athletic performance).



- Model: nominal vs nominal
  e.g. sport vs sex
  - Model or test:
    - chi-squared test or contingency table
  - Effect statistics:
    - Relative frequencies, expressed as a difference in frequencies, ratio of frequencies (relative risk), or ratio of odds (odds ratio)
    - Relative risk is appropriate for cross-sectional or prospective designs.
      - risk of having rugby disease for males relative to females is (75/100)/(30/100) = 2.5
    - Odds ratio is appropriate for case-control designs.
      - calculated as (75/25)/(30/70) = 7.0

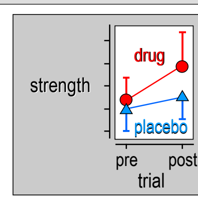


- Model: nominal vs numeric
  e.g. heart disease vs age
  - Model or test:
    - categorical modeling
  - Effect statistics:
    - relative risk or odds ratio per unit of the numeric variable (e.g., 2.3 per decade)



- Model: ordinal or counts vs whatever
  - Can sometimes be analyzed as numeric variables using regression or t tests
  - Otherwise logistic regression or generalized linear modeling
- Complex models
  - Most reducible to t tests, regression, or relative frequencies.
  - Example…

- Model: controlled trial
  (numeric vs 2 nominals)
  e.g. strength vs trial vs group
  - Model or test:
    - unpaired t test of change scores (2 trials, 2 groups)
    - repeated-measures ANOVA with within- and between-subject factors (>2 trials or groups)
    - Note: use line diagram, not bar graph, for repeated measures.
  - Effect statistics:
    - difference in change in mean expressed as raw difference, percent difference, or fraction of the pre standard deviation
  - Other statistics:
    - standard deviation representing individual responses (derived from within-subject standard deviations in the two groups)



- Model: extra predictor variable to "control for something"
  e.g. heart disease vs physical activity vs age
  - Can't reduce to anything simpler.
  - Model or test:
    - multiple linear regression or analysis of covariance (ANCOVA)
    - Equivalent to the effect of physical activity with everyone at the **same age**.
    - Reduction in the effect of physical activity on disease when age is included implies age is at least partly the **reason** or **mechanism** for the effect.
    - Same analysis gives the effect of age with everyone at same level of physical activity.
  - Can use special analysis (mixed modeling) to include a mechanism variable in a repeated-measures model. See separate presentation at newstats.org.

- Problem: some models don't fit uniformly for different subjects
  - That is, between- or within-subject standard deviations differ between some subjects.
  - Equivalently, the residuals are non-uniform (have different standard deviations for different subjects).
  - Determine by examining standard deviations or plots of residuals vs predicteds.
  - Non-uniformity makes p values and confidence limits wrong.
  - How to fix…
    - Use unpaired t test for groups with **unequal variances**, or…
    - Try taking **log** of dependent variable before analyzing, or…
    - Find some other **transformation**. As a last resort…
    - Use **rank transformation**: convert dependent variable to ranks before analyzing (= **non-parametric** analysis–same as Wilcoxon, Kruskal-Wallis and other tests).

Generalizing from a Sample to a Population

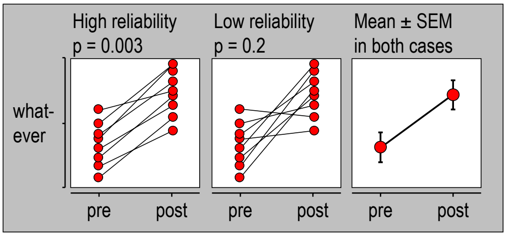
- You study a sample to find out about the population.
- The value of a statistic for a sample is only an estimate of the true (population) value.
- Express precision or uncertainty in true value using 95% confidence limits.
  - Confidence limits represent likely range of the true value.
  - They do NOT represent a range of values in different subjects.
  - There's a 5% chance the true value is outside the 95% confidence interval: the Type 0 error rate.
- Interpret the observed value and the confidence limits as clinically or practically beneficial, trivial, or harmful.
  - Even better, work out the probability that the effect is clinically or practically beneficial/trivial/harmful. See sportsci.org.

- Statistical significance is an old-fashioned way of generalizing, based on testing whether the true value could be zero or null.
  - Assume the null hypothesis: that the true value is zero (null).
  - If your observed value falls in a region of extreme values that would occur only 5% of the time, you reject the null hypothesis.
  - That is, you decide that the true value is unlikely to be zero; you can state that the result is statistically significant at the 5% level.
  - If the observed value does not fall in the 5% unlikely region, most people mistakenly accept the null hypothesis: they conclude that the true value is zero or null!
  - The p value helps you decide whether your result falls in the unlikely region.
    - If **$p<0.05$**, your result is in the unlikely region.

- One meaning of the p value: the probability of a more extreme observed value (positive or negative) when true value is zero.
- Better meaning of the p value: if you observe a positive effect, 1 - p/2 is the chance the true value is positive, and **p/2** is the chance the true value is **negative**. Ditto for a negative effect.
  - Example: you observe a 1.5% enhancement of performance (p=0.08). Therefore there is a 96% chance that the true effect is any "enhancement" and a 4% chance that the true effect is any "impairment".
  - This interpretation does not take into account trivial enhancements and impairments.
- Therefore, if you must use p values, show exact values, not $p<0.05$ or $p>0.05$.
  - **Meta-analysts** also need the exact p value (or confidence limits).

- If the true value is zero, there's a 5% chance of getting statistical significance: the Type I error rate, or rate of false positives or false alarms.
- There's also a chance that the smallest worthwhile true value will produce an observed value that is not statistically significant: the Type II error rate, or rate of false negatives or failed alarms.
  - In the old-fashioned approach to research design, you are supposed to have enough subjects to make a Type II error rate of 20%: that is, your study is supposed to have a **power** of 80% to detect the smallest worthwhile effect.
- If you look at lots of effects in a study, there's an increased chance being wrong about at least one of them.
  - Old-fashioned statisticians like to **control this inflation of the Type I error rate** within an ANOVA to make sure the increased chance is kept to 5%. This approach is misguided.

- The standard error of the mean (typical variation in the mean from sample to sample) can convey statistical significance.
  - Non-overlap of the error bars of two groups implies a statistically significant difference, but only for groups of equal size (e.g. males vs females).
  - In particular, non-overlap does NOT convey statistical significance in experiments:



- In summary
  - If you must use statistical significance, show exact p values.
  - Better still, show confidence limits instead.
  - NEVER show the standard error of the mean!
  - Show the usual between-subject standard deviation to convey the spread between subjects.
    - In population studies, this standard deviation helps convey **magnitude of differences or changes** in the mean.
  - In interventions, show also the within-subject standard deviation (the typical error) to convey precision of measurement.
    - In athlete studies, this standard deviation helps convey magnitude of differences or changes in mean **performance**.